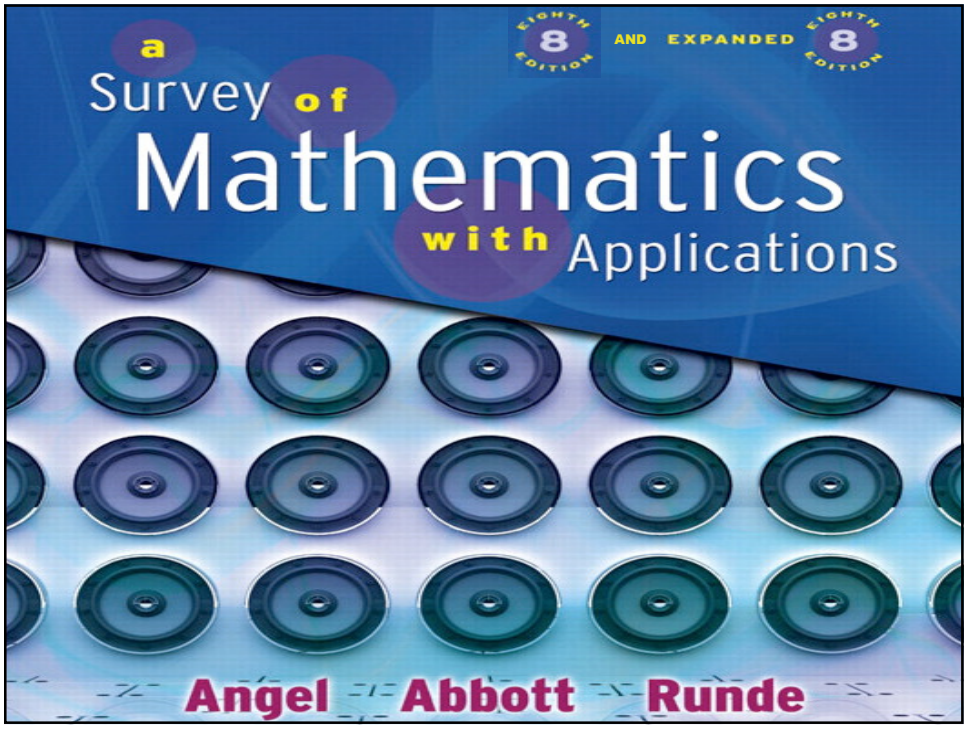

Section 6.5

Variation

Direct Variation

- Variation is an equation that relates one variable to one or more other variables.
- In *direct variation*, the values of the two related variables increase or decrease together.
- If a variable y varies directly with a variable x, then

$$y = kx$$

where k is the **constant of proportionality** (or the variation constant).

Example

- The amount of interest earned on an investment, I, varies directly as the interest rate, r. If the interest earned is \$50 when the interest rate is 5%, find the amount of interest earned when the interest rate is 7%.
- $I = kr$

  $50 = k(0.05)$

  $1000 = k$

Example (continued)

- $k = 1000$, $r = 7\%$

  $I = kr$

  $I = 1000(0.07)$

  $I = 70$
- The amount of interest earned is $70.

Inverse Variation

- When two quantities vary inversely, as one quantity increases, the other quantity decreases, and vice versa.
- If a variable y varies inversely with a variable, x, then

$$y = \frac{k}{x}$$

where k is the constant of proportionality.

Example

- Suppose y varies inversely as x. If $y = 12$ when $x = 18$, find y when $x = 21$.

$$y = \frac{k}{x}$$

$$12 = \frac{k}{18}$$

$$216 = k$$

- Now substitute 216 for k, and find y when $x = 21$.

$$y = \frac{k}{x}$$

$$y = \frac{216}{21}$$

$$y = 10.3$$

Joint Variation

- One quantity may vary directly as the product of two or more other quantities.

- The general form of a joint variation, where y, varies directly as x and z, is

$$y = kxz$$

where k is the constant of proportionality.

Example

- The area, A, of a triangle varies jointly as its base, b, and height, h. If the area of a triangle is 48 in^2 when its base is 12 in. and its height is 8 in., find the area of a triangle whose base is 15 in. and whose height is 20 in.

$$A = kbh$$
$$48 = k(12)(8)$$
$$48 = k(96)$$
$$k = \frac{48}{96} = \frac{1}{2}$$

$$A = kbh$$
$$A = \frac{1}{2}(15)(20)$$
$$A = 150 \text{ in.}^2$$

Combined Variation

- A varies jointly as B and C and inversely as the square of D. If $A = 1$ when $B = 9$, $C = 4$, and $D = 6$, find A when $B = 8$, $C = 12$, and $D = 5$.
- Write the equation.

$$A = \frac{kBC}{D^2}$$

Combined Variation (continued)

- Find the constant of proportionality.

$$A = \frac{kBC}{D^2}$$

$$1 = \frac{k(9)(4)}{6^2}$$

$$1 = \frac{36k}{36}$$

$$1 = k$$

- Now find A.

$$A = \frac{kBC}{D^2}$$

$$A = \frac{(1)(8)(12)}{5^2}$$

$$A = \frac{96}{25}$$

$$A = 3.84$$